

07 | Linear Motion

Along a straight line the whole story of a motion fits in three graphs and four equations. The slope of a graph is the next quantity up, the area under it is the quantity before, and free fall is just constant acceleration with $a = g$.

1. Position-Time Graphs · 2. Velocity-Time Graphs · 3. Equations of Motion · 4. Vertical Motion · 5. Exam Focus
· 6. Challenge Zone

Key terms 關鍵詞 position 位置 · displacement-time graph 位移時間圖 · velocity-time graph 速度時間圖 · uniform acceleration 勻加速度 · deceleration 減速度 · free fall 自由下墜 · acceleration due to gravity 重力加速度 · air resistance 空氣阻力 · terminal velocity 終端速度

1 Position-Time Graphs

• CONCEPT Describing a straight-line motion with signs

- For motion along a line, pick an origin and a positive direction.
- The **position** s is the signed distance from the origin, and the **displacement** is the change in position, $\Delta s = s_2 - s_1$.
- Velocity and acceleration are also signed: a negative velocity just means moving in the negative direction.

On an s - t graph:

- the **slope** is the velocity: $v = \frac{\Delta s}{\Delta t}$, and at an instant it is the slope of the tangent;
- a **straight line** means constant velocity, a **horizontal line** means at rest, a line sloping down means moving in the negative direction;
- a **curve** means the velocity is changing: steepening means speeding up, flattening means slowing down.

• WORKED EXAMPLE Reading a position-time graph

- Fig 7.1 shows the position of a trolley on a track.
- Find (a) its velocity in each stage, (b) its displacement and distance over the 9 s, (c) its average velocity and average speed.

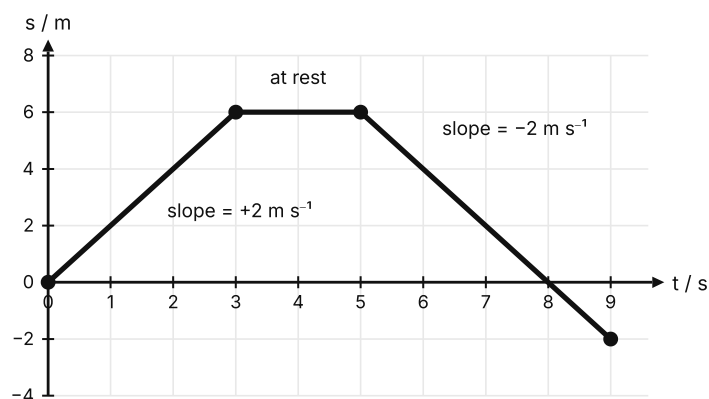


Fig 7.1 A position-time graph: read velocities from the slopes

- **(a)** 0 to 3 s: $\frac{6-0}{3} = +2 \text{ m s}^{-1}$. 3 to 5 s: 0 (at rest). 5 to 9 s: $\frac{-2-6}{4} = -2 \text{ m s}^{-1}$.
- **(b)** Displacement = $-2 - 0 = -2 \text{ m}$.
- Distance = $6 + 0 + 8 = 14 \text{ m}$ (it went forwards 6 m and then backwards 8 m).
- **(c)** Average velocity = $-2/9 = -0.22 \text{ m s}^{-1}$; average speed = $14/9 = 1.6 \text{ m s}^{-1}$.

• WATCH OUT **Three traps with position-time graphs**

- The graph is **not** a picture of the path. A line sloping downwards does not mean going downhill; it means the position value is decreasing.
- A horizontal line is standing still, not moving at constant speed.
- The value read from the axis is a **position**. Only the difference between two readings is a displacement, and distance needs the turning points.

• TRY IT

- Q1** For an s - t graph, say what each means: (a) the slope, (b) a horizontal line, (c) a curve that gets steeper with time, (d) a straight line sloping downwards. · Basic
- Q2** A lift is at $s = 12$ m at $t = 2$ s and at $s = 3$ m at $t = 5$ s, moving at constant velocity. Find its velocity and describe the motion. · Basic
- Q3** A runner leaves a line at 4 m s^{-1} for 5 s, rests for 3 s, then jogs back to the line at 2 m s^{-1} . Describe the s - t graph (its key points) and find the average speed and the average velocity for the whole trip. · Medium

2 Velocity-Time Graphs

• CONCEPT **Slope is acceleration, area is displacement**

On a v - t graph:

- the **slope** is the acceleration: $a = \frac{\Delta v}{\Delta t}$;
- the **area between the graph and the time axis** is the displacement. Area above the axis counts as positive, area below as negative;
- where the graph crosses the axis, the object is momentarily at rest and turns round;
- a horizontal line is uniform motion, a sloping straight line is uniform acceleration.
- The same pattern repeats one level down: the area under an a - t graph is the change in velocity.

○ WHY IT WORKS **Why the area gives the displacement**

- Cut the time axis into strips of width Δt so short that the velocity is almost constant in each.
- During one strip the displacement is $v \Delta t$, which is exactly the area of that thin rectangle.
- Adding all the strips gives the total displacement, which is the total area.
- For a straight-line graph the shape is a triangle or trapezoid, so its area is easy to find.

• **WORKED EXAMPLE A journey with three stages**

- Fig 7.2 shows the speed of a car.
- Find the acceleration in each stage, the distance travelled, and the average speed.

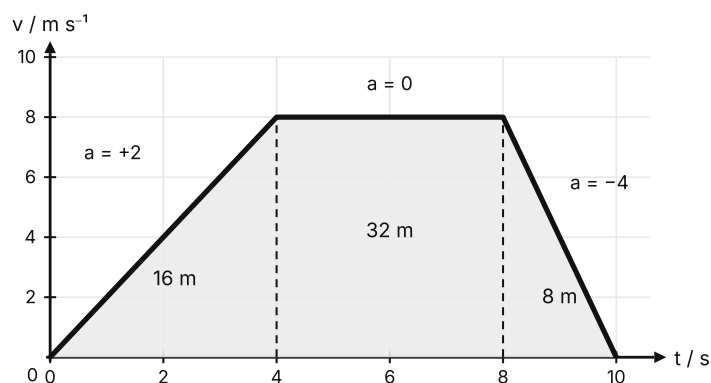


Fig 7.2 Slopes give the accelerations; the three areas add up to the distance

- Stage 1 (0 to 4 s): $a = \frac{8-0}{4} = +2 \text{ m s}^{-2}$, area = $\frac{1}{2} \times 4 \times 8 = 16 \text{ m}$.
- Stage 2 (4 to 8 s): $a = 0$, area = $4 \times 8 = 32 \text{ m}$.
- Stage 3 (8 to 10 s): $a = \frac{0-8}{2} = -4 \text{ m s}^{-2}$, area = $\frac{1}{2} \times 2 \times 8 = 8 \text{ m}$.
- Distance = $16 + 32 + 8 = 56 \text{ m}$ in 10 s, so the average speed is 5.6 m s^{-1} .
- The car never reverses, so this is also the displacement.

• **CONCEPT The same motion in three graphs**

- The three graphs of one motion are linked by slope and area.
- Fig 7.3 shows the car of Fig 7.2: a constant acceleration gives a straight $v-t$ line and a parabola on $s-t$; constant velocity gives a straight $s-t$ line; slowing down makes the $s-t$ curve flatten.

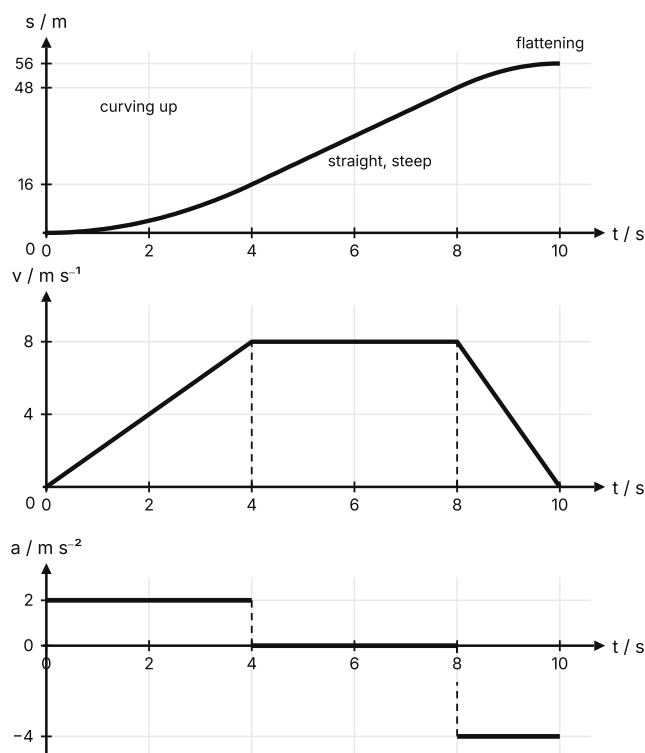


Fig 7.3 Reading from the bottom graph up: area gives the next graph, slope gives the previous one

• WATCH OUT **Common errors with velocity-time graphs**

- The slope of a v - t graph is the **acceleration**, not the velocity. A horizontal line at 8 m s^{-1} means $a = 0$.
- Where two v - t graphs cross, the two objects have **equal velocities**. That is the moment of largest or smallest gap, not the moment they meet. They meet when the **areas** (displacements from a common start) are equal (2015 P1A Q9).
- Area below the axis is negative displacement. Add it as distance but subtract it for displacement.
- The graph shows how fast, not where. Two objects with the same v - t graph can be in different places.

• TRY IT

- Q4** A v - t graph is a straight line from $(0, 4 \text{ m s}^{-1})$ to $(5 \text{ s}, 14 \text{ m s}^{-1})$. Find the acceleration and the displacement in the 5 s. · Basic
- Q5** The velocity of a trolley decreases uniformly from $+6 \text{ m s}^{-1}$ at $t = 0$ to -2 m s^{-1} at $t = 4 \text{ s}$. Find (a) the acceleration, (b) the instant it is momentarily at rest, (c) its displacement and the distance it travels in the 4 s. · Medium
- Q6** A sprinter accelerates uniformly from rest to 10 m s^{-1} in 2 s and then runs at that speed. How far does she run in 10 s, and how long does she take to run 100 m? · Medium

• PRACTICE

- Q7** A lift accelerates uniformly from rest to 2.4 m s^{-1} in 3 s, moves at 2.4 m s^{-1} for 5 s, then slows uniformly to rest in 2 s. Find the acceleration in each stage, the height gained and the average velocity. · Medium
- Q8** A car starts from rest, accelerates at 2 m s^{-2} for 5 s, moves at constant speed for 10 s, then brakes at 5 m s^{-2} to a stop. Sketch the v - t graph and find the total distance and the average speed. · Harder

3 Equations of Motion

• CONCEPT **Uniformly accelerated motion in a straight line**

- When the acceleration is **constant** and the motion is along a line, five quantities are linked: the initial velocity u , final velocity v , acceleration a , displacement s and time t .
- Give any three and the other two follow from the equations below.

• FORMULA **The four equations**

$$v = u + at \quad s = \frac{1}{2}(u + v)t \quad s = ut + \frac{1}{2}at^2 \quad v^2 = u^2 + 2as$$

Variable not involved	Use
s	$v = u + at$
a	$s = \frac{1}{2}(u + v)t$
v	$s = ut + \frac{1}{2}at^2$
t	$v^2 = u^2 + 2as$

○ WHY IT WORKS **Where the equations come from**

- Fig 7.4 shows a constant acceleration: the velocity rises steadily from u to v in time t .

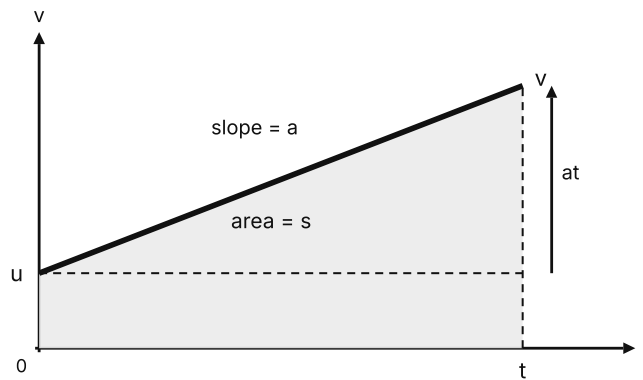


Fig 7.4 The slope of the line is a and the area of the trapezoid is s

- **Slope.** $a = \frac{v - u}{t}$, so $v = u + at$.
- **Area.**
- The trapezoid has parallel sides u and v and width t , so $s = \frac{1}{2}(u + v)t$.
- The average velocity is $\frac{1}{2}(u + v)$ only because the graph is a straight line.
- **Eliminate v .**
- Put $v = u + at$ into the area: $s = \frac{1}{2}(2u + at)t$, so $s = ut + \frac{1}{2}at^2$.
- **Eliminate t .**
- Put $t = \frac{v - u}{a}$ into $s = \frac{1}{2}(u + v)t$: $s = \frac{(u + v)(v - u)}{2a} = \frac{v^2 - u^2}{2a}$, so $v^2 = u^2 + 2as$.

• CONCEPT **A method that always works**

1. Choose the positive direction (usually the direction of the initial velocity) and stick to it.
2. List u , v , a , s , t with signs, writing "?" for the unknown. "Starts from rest" means $u = 0$. "Comes to rest" means $v = 0$.
3. Pick the equation that does **not** contain the quantity you neither know nor want.
4. Solve, then check the sign and size make sense.

• WORKED EXAMPLE **Braking a car**

- A car slows from 20 m s^{-1} to 8 m s^{-1} while travelling 42 m along a straight road.
- Find its acceleration and the time taken.
- $u = 20$, $v = 8$, $s = 42$; unknown a , t .
- No t is involved in $v^2 = u^2 + 2as$:

$$a = \frac{v^2 - u^2}{2s} = \frac{64 - 400}{84} = -4.0 \text{ m s}^{-2}$$

- The negative sign means deceleration.
- Then $t = \frac{v - u}{a} = \frac{8 - 20}{-4.0} = 3.0 \text{ s}$.

• WORKED EXAMPLE **Stopping distance**

- A driver travelling at 72 km h^{-1} sees a hazard.
- The reaction time is 0.6 s and the car then brakes at 5.0 m s^{-2} .
- Find the stopping distance, and the distance if the speed were doubled.
- $72 \text{ km h}^{-1} = 20 \text{ m s}^{-1}$.
- **Thinking distance** (no braking yet, constant speed): $20 \times 0.6 = 12 \text{ m}$.
- **Braking distance**: $0 = 20^2 + 2(-5.0)s$, so $s = 40 \text{ m}$.
- Stopping distance = 52 m .
- At 40 m s^{-1} : thinking = 24 m , braking = $\frac{40^2}{2 \times 5.0} = 160 \text{ m}$, total 184 m .
- Doubling the speed doubles the thinking distance but **quadruples** the braking distance, because $s \propto v^2$.

• WORKED EXAMPLE **Motion in successive stages**

- A lift rises 30 m from rest to rest.
- It accelerates at 1.5 m s^{-2} up to 3.0 m s^{-1} , then moves at that speed, and finally slows at 1.0 m s^{-2} .
- How long does the trip take?
- Speeding up: $t_1 = \frac{3.0}{1.5} = 2.0 \text{ s}$ and $s_1 = \frac{1}{2}(3.0)(2.0) = 3.0 \text{ m}$.
- Slowing down: $t_3 = \frac{3.0}{1.0} = 3.0 \text{ s}$ and $s_3 = \frac{1}{2}(3.0)(3.0) = 4.5 \text{ m}$.
- The steady part covers $30 - 3.0 - 4.5 = 22.5 \text{ m}$ in $\frac{22.5}{3.0} = 7.5 \text{ s}$.
- Total time = $2.0 + 7.5 + 3.0 = 12.5 \text{ s}$.

• WATCH OUT **Equation traps**

- The four equations need a **constant** acceleration in a **straight line**. If the acceleration changes, split the motion into stages.
- Deceleration is a negative a when u is positive. Keep the signs consistent, or the answers come out wrong.
- s is a **displacement**. If the object turns round (a ball thrown up and returning), s is not the distance travelled.
- Change km h^{-1} to m s^{-1} first. Divide by 3.6 .

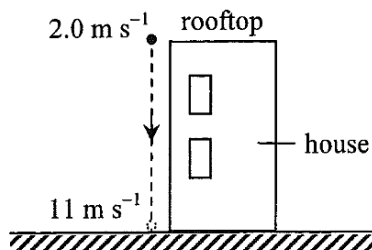
• TRY IT

- Q9** A car starts from rest and accelerates uniformly at 2.5 m s^{-2} for 8.0 s . Find its final speed and the distance it travels. · Basic
- Q10** A train slows uniformly from 30 m s^{-1} to 12 m s^{-1} in 6.0 s . Find its acceleration and the distance travelled. · Medium
- Q11** After a crash the police measure skid marks 30 m long. On this road a car brakes at 8.0 m s^{-2} . Was the car travelling faster than the 60 km h^{-1} limit before braking? · Medium
- Q12** An object moving at 5.0 m s^{-1} accelerates uniformly at 3.0 m s^{-2} over a distance of 20 m . Find its final speed and the time taken. · Medium

• PRACTICE

- Q13** An object moves along a line with velocity $v = 6 - 2t$ (in m s^{-1} , with t in s). Find (a) the time at which it stops, (b) its greatest distance from the start, (c) when it returns to the start, (d) the total distance travelled in the first 6 s . · Harder
- Q14** A speeding car passes a stationary police car at 24 m s^{-1} . At that instant the police car starts from rest with acceleration 3.0 m s^{-2} . When and where does it catch the car, and how fast is it then travelling? · Harder

9.

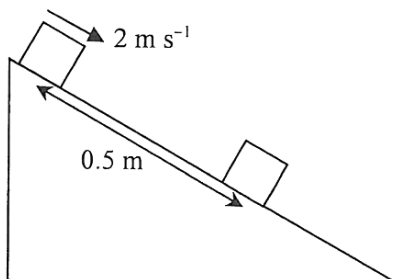


A particle is projected vertically downward with an initial speed of 2.0 m s^{-1} from the rooftop of a house. The particle reaches the ground with a speed of 11 m s^{-1} as shown. Estimate the height of the house. Neglect air resistance. ($g = 9.81 \text{ m s}^{-2}$)

- A. 3.3 m
- B. 6.0 m
- C. 6.5 m
- D. 12 m

Working / answer:

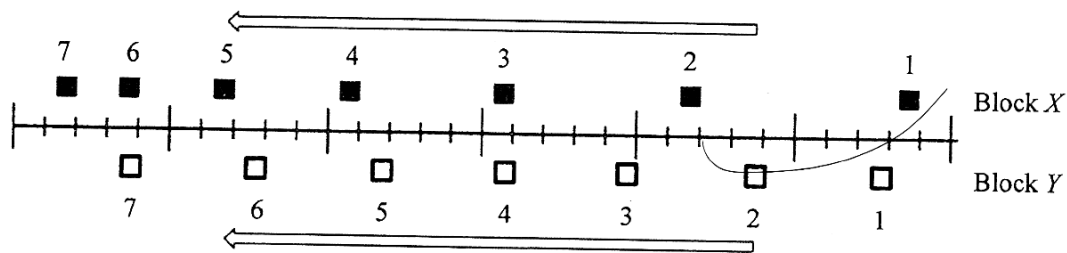
7. A block with initial speed 2 m s^{-1} slides down a rough inclined plane and stops after travelling a distance of 0.5 m . What is the deceleration of the block?



- A. 1 m s^{-2}
- B. 2 m s^{-2}
- C. 4 m s^{-2}
- D. Answer cannot be found as the angle of inclination of the plane is not given.

Working / answer:

7. Two blocks X and Y are moving towards the left. Their positions at successive instants (indicated by the numbers) of equal time intervals are shown below.

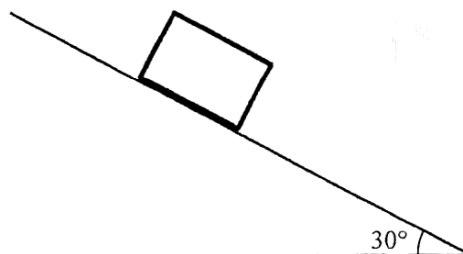


Do these two blocks ever have the same instantaneous speed ?

- A. Yes, at instant 3.
- B. Yes, at a moment between instants 4 and 5.
- C. Yes, at instant 6.
- D. No.

Working / answer:

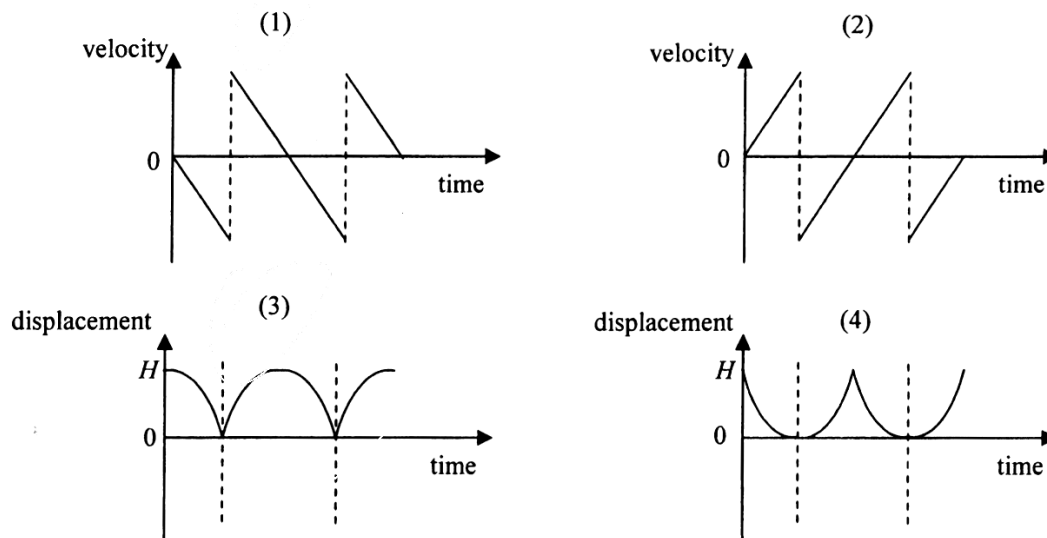
6. A block is released from rest on an inclined plane as shown. The inclined plane makes an angle of 30° to the horizontal. The block moves with uniform acceleration, and travels a distance of 1 m in the first 3 s. Determine the acceleration of the block.



- A. 0.22 m s^{-2}
- B. 0.33 m s^{-2}
- C. 4.91 m s^{-2}
- D. Cannot be determined as the frictional force acting on the block is unknown.

Working / answer:

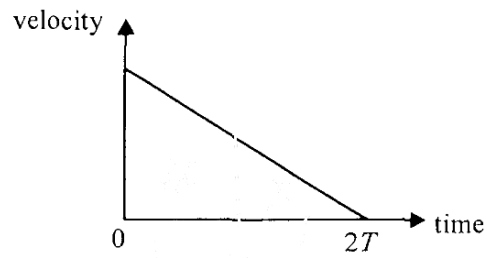
7. Which of the following graphs (velocity-time and displacement-time) best represent the motion of a ball falling from rest under gravity at a height H and bouncing back from the ground two times? Assume that the collision with the ground is perfectly elastic and neglect air resistance. (Downward measurement is taken to be negative.)



- A. (1) and (3) only
 B. (1) and (4) only
 C. (2) and (3) only
 D. (2) and (4) only

Working / answer:

7. At time $t = 0$, a small sphere is projected up along a smooth incline with a certain initial velocity. It travels a distance L and becomes momentarily at rest after a time $2T$. The corresponding velocity-time graph is shown below.



What is the distance travelled by the sphere from $t = 0$ to $t = T$?

- A. $\frac{1}{4}L$
B. $\frac{1}{2}L$
C. $\frac{3}{4}L$
D. $\frac{4}{5}L$

Working / answer:

4 Vertical Motion

• CONCEPT Free fall: the same acceleration for everything

- An object moving under the pull of gravity alone is in **free fall**.
- It has the same downward acceleration whatever its mass, the **acceleration due to gravity** $g = 9.81 \text{ m s}^{-2}$ near the Earth's surface.
- This holds for objects moving up, down or momentarily at rest.
- A coin and a feather dropped together in air land at different times only because of **air resistance**, which affects the light, spread-out feather much more.
- In a vacuum tube they fall side by side.
- Measurements of g use a falling object with a ticker-timer, light-gates or video analysis.
- Air resistance grows with speed.
- A falling object therefore accelerates less and less until air resistance equals its weight; then the acceleration is zero and the speed stays at the **terminal velocity** (Fig 7.6, right).

• FORMULA Up-and-down motion (take upwards as positive)

- Use the four equations with $a = -g$ and u the launch velocity:

$$v = u - gt \quad s = ut - \frac{1}{2}gt^2 \quad v^2 = u^2 - 2gs$$

- For a ball launched upwards at u from ground level: time to the top $T = \frac{u}{g}$, maximum height $H = \frac{u^2}{2g}$, total time in the air $2T = \frac{2u}{g}$, and it returns with speed u .

○ WHY IT WORKS Why up and down look symmetric

- At the top $v = 0$, so $0 = u - gT$ gives $T = u/g$ and $0 = u^2 - 2gH$ gives $H = u^2/2g$.
- Coming down from rest, the fall of height H takes $\sqrt{2H/g} = u/g = T$, the same time as the rise, and the speed at the bottom is $\sqrt{2gH} = u$.
- Fig 7.5 shows why: the v - t line has one slope $-g$ throughout, and the two triangles above and below the axis are congruent.

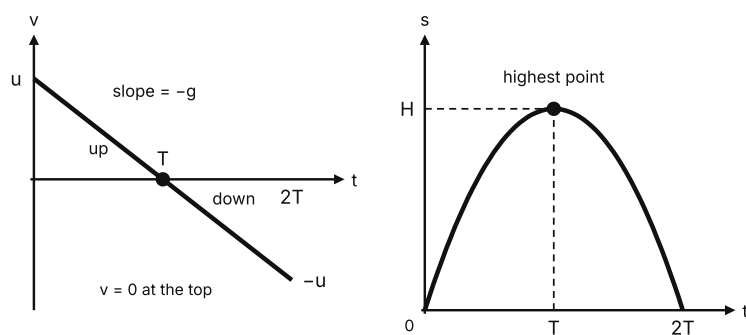


Fig 7.5 A ball thrown upwards: the velocity falls steadily, crossing zero at the highest point; the position graph is a parabola

• **CONCEPT Bouncing balls and air resistance on the graphs**

- A ball dropped from rest and bouncing elastically has a sawtooth v - t graph (upwards positive): a straight line of slope $-g$, with an instant reversal of velocity at each bounce.
- The acceleration during flight is always $-g$, with a brief large upward acceleration at each bounce.
- With air resistance the v - t line for a falling object bends over towards the terminal velocity.

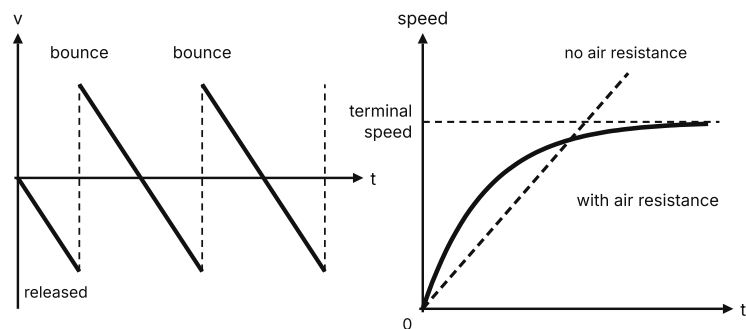


Fig 7.6 Left: a ball bouncing elastically (2012 P1A Q7); every sloping line has slope $-g$. Right: falling with and without air resistance

• **WORKED EXAMPLE Throwing a ball from a building**

- A ball is thrown vertically upwards at 15 m s^{-1} from the top of a 20 m building.
- Take $g = 9.81 \text{ m s}^{-2}$.
- Find (a) the greatest height above the top, (b) the time to return to the level of the top, (c) the speed and time when it reaches the ground.
- **(a)** $H = \frac{u^2}{2g} = \frac{225}{19.62} = 11.5 \text{ m}$. **(b)** $2u/g = \frac{30}{9.81} = 3.06 \text{ s}$.
- **(c)** The ground is at $s = -20 \text{ m}$. $v^2 = 15^2 + 2(9.81)(20) = 617.4$, so the speed is $\sqrt{617.4} \approx 24.8 \text{ m s}^{-1}$, directed downwards.
- Keeping the unrounded velocity, $t = (15 + \sqrt{617.4})/9.81 \approx 4.06 \text{ s}$.

• **WATCH OUT Free-fall traps**

- At the highest point $v = 0$ but $a = g$ downwards, not zero. The ball has not "run out of acceleration".
- g does not depend on mass. A heavier stone does not fall faster (apart from air resistance).
- Decide the positive direction first. If upwards is positive, $a = -g$ and a ball falling below the start has negative s .
- With air resistance, the equations with $a = -g$ no longer apply. Only the graph shapes do.
- Use $g = 9.81 \text{ m s}^{-2}$ unless the question gives another value.

• **TRY IT**

Q15 A stone is dropped from rest. Find its speed and the distance fallen after 3.0 s. Neglect air resistance. · Basic

Q16 A ball is thrown vertically upwards at 12 m s^{-1} from the ground. Find the greatest height reached and the time taken to return to the ground. · Medium

Q17 A ball is dropped from 5.0 m onto the floor and rebounds to a height of 3.2 m. Find its speed just before and just after the impact. · Medium

• **PRACTICE**

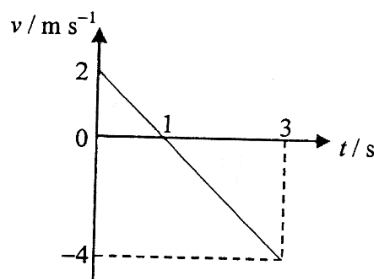
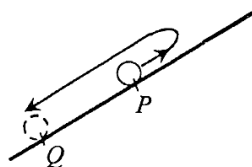
Q18 Ball A is dropped from the top of a 45 m tower. One second later ball B is thrown vertically downwards from the same point and the two balls reach the ground together. Find the initial speed of B. · Harder

10. A train departs from one station and stops at the next station after travelling 1.2 km along a straight line. The maximum acceleration and deceleration of the train are both 5 m s^{-2} and the highest speed of the train is limited to 20 m s^{-1} . Find the shortest time taken for this trip.

- A. 56 s
 B. 58 s
 C. 62 s
 D. 64 s

Working / answer:

11. A bead is projected up along a smooth incline at point P at time $t = 0$. After reaching the highest point, the bead then travels downwards and passes point Q at $t = 3 \text{ s}$ as shown. The velocity-time (v - t) relationship of the bead is shown in the graph below. Find the separation PQ along the incline.



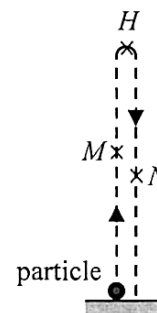
- A. 2 m
 B. 3 m
 C. 4 m
 D. 5 m

Working / answer:

6. A particle is thrown vertically upward and its path is shown below. H is the highest point that the particle reached. Which of the following statements about the particle is/are correct? Neglect air resistance.

- (1) Its acceleration at M is upward.
 (2) Its acceleration at H is zero.
 (3) Its acceleration at N is downward.

- A. (1) only
 B. (3) only
 C. (2) and (3) only
 D. (1), (2) and (3)



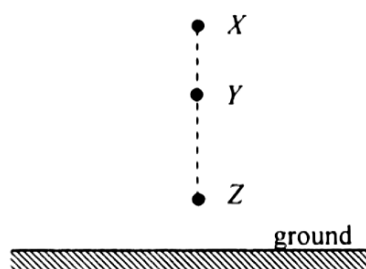
Working / answer:

5. A particle is moving along a straight line with uniform acceleration. It takes 4 s to travel a distance of 36 m and then 2 s to travel the next 36 m. What is its acceleration?

- A. 2.5 m s^{-2}
 B. 3.0 m s^{-2}
 C. 4.0 m s^{-2}
 D. 4.5 m s^{-2}

Working / answer:

- 8.

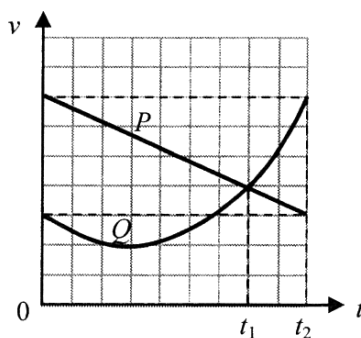


A particle is released from rest at X as shown. It takes time t_1 to fall from X to Y and time t_2 to fall from Y to Z . If $XY : YZ = 9 : 16$, find $t_1 : t_2$. Neglect air resistance.

- A. 2 : 3
 B. 3 : 4
 C. 4 : 3
 D. 3 : 2

Working / answer:

9.



The figure shows the velocity-time (v - t) graph of two cars P and Q travelling along the same straight road. At $t = 0$, the cars are at the same position. Which deductions about the cars between $t = 0$ and $t = t_2$ are correct?

- (1) P and Q are always travelling in the same direction.
- (2) At $t = t_1$, the separation between P and Q is at a maximum.
- (3) At $t = t_2$, Q lags behind P .

- A. (1) and (2) only
- B. (1) and (3) only
- C. (2) and (3) only
- D. (1), (2) and (3)

Working / answer:

A stone falls from rest. Neglecting air resistance, the ratio of the distance travelled by the stone in the first second to that traveled in the second second is

- A. 1 : 1.
- B. 1 : 2.
- C. 1 : 3.
- D. 1 : 4.

Working / answer:

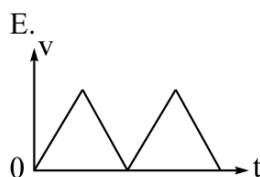
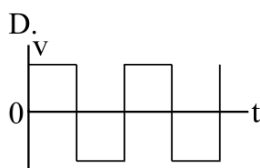
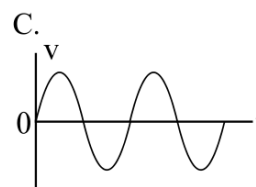
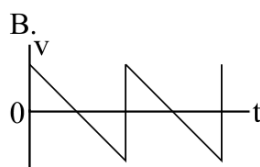
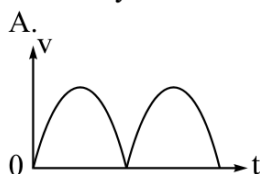
Two bodies are allowed to fall freely from the same location, one of which is released a short time before the other. If the air resistance can be neglected, while they are falling

- (1) the two bodies have the same acceleration.
- (2) their speeds always differ by the same amount.
- (3) their distance of separation is always the same.

- A. If (1), (2) and (3) are all correct
- B. If (1) and (2) only are correct
- C. If (2) and (3) only are correct
- D. If (1) only is correct
- E. If (3) only is correct

Working / answer:

A ball bounces up and down from the floor. Which of the following graphs shows the variation of its velocity v with time t ?

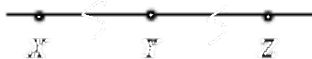


Working / answer:

Two small identical objects P and Q are released from rest from the top of a building 80 m above the ground. Q is released 1 s after P . Neglecting air resistance, what is the maximum vertical separation between P and Q in the air?

- A. 5 m
- B. 10 m
- C. 35 m
- D. 45 m

Working / answer:



An object accelerates uniformly along a straight line from X to Z . It passes X and Z with speeds u and v respectively. What is its speed when it passes Y , which is the mid-point of XZ ?

- A. $\sqrt{\frac{u^2 + v^2}{2}}$
- B. $\sqrt{uv}$
- C. $\frac{2uv}{u+v}$
- D. $\frac{u+v}{2}$
- E. It cannot be found as the distance between X and Z is not known.

Working / answer:

An object falls from rest from the top of tower. The object takes 0.4 s to fall from the mid-point of the tower to the ground. Calculate the time taken for the object to fall from the top of the tower to the ground. (Neglect air resistance.)

- A. 0.68 s
- B. 0.80 s
- C. 1.20 s
- D. 1.37 s

Working / answer:

5 Exam Focus

• CONCEPT How the marks are earned in this chapter

Question type	Seen in	What earns the mark
Reading s - t and v - t graphs	2018 Q7, 2021 Q7, 2018 Q11, 2015 Q9	State what the slope or area means, then use it. Signed area for displacement; equal velocities where v - t graphs cross, not meeting.
One equation, three knowns	2014 Q5, 2016 Q7, 2017 Q6, 2014 Q9	List u , v , a , s , t , choose the equation with no unwanted variable. Full marks need the correct sign and unit.
Two-stage or shortest-time problems	2018 Q10	Split into speeding up, steady, slowing down. Do not assume the top speed is held the whole way.
Free fall and gravity graphs	2013 Q8, 2020 Q6, 2012 Q7	Acceleration is g at every point (including the top). From rest, $s \propto t^2$ gives time ratios.

- In Paper 1B the same skills appear as "find the acceleration from the v - t graph, then the distance from the area", followed by a reason why the real graph differs (air resistance, friction).
- Show the graph reading in the working.

6 Challenge Zone

• CHALLENGE Beyond the exam: extend the ideas

- C1 Catching up.** (*inspired by 2015 P1A Q9*) Car P moves at a constant 12 m s^{-1} . Car Q starts from rest at the same point at $t = 0$ and accelerates at 2.0 m s^{-2} . (a) When does Q catch P? (b) When is the gap largest, and how large is it? (c) Sketch both v - t graphs and explain why the graphs cross at (b) but not at (a).
Hint: the gap is the difference of the two areas.
- C2 The last second.** A stone falls from rest and in the last second of its fall it drops 45 m. Take $g = 10 \text{ m s}^{-2}$ and neglect air resistance. Find the total time of fall and the height of the drop.
Hint: write "height fallen in T s" minus "height fallen in $T - 1$ s".
- C3 Stone from a balloon.** A balloon rises at a steady 5.0 m s^{-1} . When it is 30 m above the ground a stone is released from it. How long does the stone take to reach the ground?
Hint: at release the stone shares the balloon's upward velocity.
- C4 The midpoint speed.** (*adapted from HKALE 2001 Paper IIA Q5*) An object moving with constant acceleration passes points X and Z with speeds u and v . Prove that its speed at the midpoint Y of XZ is $\sqrt{\frac{u^2 + v^2}{2}}$, and show that this is larger than $\frac{u+v}{2}$ unless $u = v$.
Hint: apply $v^2 = u^2 + 2as$ from X to Y and from X to Z.
- C5 Air resistance.** A 0.50 kg ball falls with a resistive force proportional to its speed and reaches a terminal speed of 20 m s^{-1} . Find the resistive force and the acceleration when its speed is 10 m s^{-1} . Sketch the a - t graph.
Hint: at terminal speed the resistive force equals the weight.